THE NEW MATHEMATICS OF ARCHITECTURE

JANE BURRY + MARK BURRY

THE NEW MATHEMATICS OF ARCHITECTURE

with 628 illustrations, 435 in colour

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This book is dedicated to John E. Pallett

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THE PARADOXICAL MATHEMATICS OF CONTEMPORARY ARCHITECTURE

Brett Steele

The mathematical phenomenon always develops out of simple arithmetic, so useful in everyday life, out of numbers, those weapons of the gods: the gods are there, behind the wall, at play with numbers.

LE CORBUSIER

It has long been said that architecture is a game played with clear objectives, but no guiding set of rules. Mathematics, on the other hand, has forever been described by its believers as a form of knowledge best understood as a game with lots of rules, but no clear objective. For evidence of the enduring beauty of this paradoxical combination of two distinct (intertwined, even if opposing) human endeavours, look no further than this wonderful book, which Jane and Mark Burry have edited as an invaluable contribution to mathematics in the architecture of our time.

Early on in the 20th century, Le Corbusier observed that, however different or new modern architecture was, mathematics was still at its heart. It seemed an obvious point for the Swiss master to make then, and one no more (or less) surprising today, when we remember that architects have needed mathematics since at least the time when one of their kind drew a right angle with a stick in the sand and realized numbers went a long way to help reliably communicate its idea to someone else (and recall, this is what architects most fundamentally do: they communicate ideas – instructions – to others). That mathematics is an important part of architecture is pretty obvious a point to a field of enquiry whose own origins lie with the publication of Vitruvius' Ten Books some two thousand years ago, made up as it is by a series of (numerically ordered) universal formulae and mathematical equations explaining how architects might best arrange architectural matter in meaningful, lasting ways. In other words, to say that mathematics is integral to architecture is like saying numbers are helpful when trying to count.

So what's the big deal about architecture and mathematics conversing in the ways they do today? Plenty, I'd say, without one having to judge a book by (the title on) its cover. The most surprising feature of a recent and evolutionary leap in Le Corbusier's 'weapons of the gods' is the simultaneity of, on the one hand, a growing power and complexity of mathematical processes in architecture, and, on the other, a marked disappearance of numbers themselves within its own language and discourses. Architects no longer speak the language, as did their ancestors, of whole numbers or predictable geometries, any more than they do of drawings made at fixed dimension or measurable scales. This situation owes itself to another kind of contemporary revolution of course: architects' near-universal assimilation of digital, information-based design platforms in their studios, which have now become the basis for not only architectural practice, but, as well, the very communication media through which their ideas are now conceived, flow and proliferate.

In considering the ubiquity of computing and information

systems within design studios today, the pervasiveness of mathematical processing within this regime is often overlooked, most often owing to the design of programmes whose appearance deliberately seeks to conceal their (higher-order) mathematical and logical composition. Consider, for example, even the most basic or traditional of architectural design activities; say, the simple drawing of a line. Within a digital modelling environment, this activity (itself either a sequence of clicks or keyboard strokes) now invokes an astonishing series of logical and mathematical operations of stunning complexity, undertaken at unimaginable speeds, in order for a programme to do something as simple as displaying that line to an architect for his or her further contemplation.

And this is where things have suddenly gotten very interesting; at least, regarding a contemporary re-animation of the relationship between architecture and mathematics. From the time of ancient Vitruvian geometric ideals to modern Corbusian regulating lines and Miesian modular grids, architecture has always been bound to (if not by) a conscious use of numbers. As a consequence, architects of all stripes have invariably embedded the language of numbers and mathematics into their own language of architecture, something that the past several centuries of architectural treatises readily confirms. A recent and decided disappearance of literal numbering or geometric descriptions (now concealed within software modelling environments that keep this information behind what a user sees on a software interface) has, accordingly, left a very real void in the language of architecture. And this is where the projects that follow offer such a valuable contribution: not only in the accomplishments of their form, beauty or material realization, but also their willingness to take on directly the challenges of reinventing the very language – and numbers – lying hidden behind their surface.

MATHEMATICS AND DESIGN

INTRODUCTION

We have reached the end of a decade and a half in which digital computation has given architects new creative opportunities with which to access the geometrical space opened up by post-17th-century mathematicians. The resulting new wave of interest in the relationship of mathematics to space-making has been aesthetically driven, and yet its expression has transcended the metaphorical. It has found expression from within the process of making as a new species of architecture, and has infiltrated architectural process in ways that have forged radical change. This book is an account of the ways in which this new mathematical focus has manifested in designed and built projects since the mid-1990s. Had it been written ten years earlier, the phenomenon would have been fresh and urgent, but the book itself would have been much more theoretical and speculative. And yet there is still a sense of being at the cusp of a much greater revolution in the representation and production of architecture to which this deep interest in the beauty of the mathematical idea has contributed.

The New Mathematics of Architecture has no pretension to present a comprehensive, or even cursory, history of the complex and multiple relationships between mathematics and architecture. They have always been very closely related; both have their deepest roots embedded in geometry. Architecture has been concerned with the creation of space; mathematics with its description and definition. In mathematics particularly, this has encompassed increasingly diverse and abstract kinds of space. The terms 'mathematics' and 'geometry' are not synonymous. Auguste Comte wrote of mathematics in 1851 that 'the plural form of the name (grammatically used as singular) indicates the want of unity in its philosophical character, as commonly conceived'.[1] Mathematics is gathered from different activities. Geometry was one of the seven liberal arts, belonging to the Quadrivium, along with arithmetic, astronomy and music (the additional Trivium included grammar, rhetoric and dialectic). The 'liberal' in liberal arts implied the study of subjects that, unlike architecture, were not necessarily directed to a profession. To geometry and arithmetic add algebra, and you have the basis of mathematics as it has developed since the Renaissance. But it is clear that these three disciplines, while closely related, developed in response to different impulses and practical needs.

The mathematically thematic chapters that follow, and into which the architectural projects have been grouped, are concerned in part with geometry in architecture. In particular, they are concerned with the embrace by architecture of geometry's expanded definition to the multiple geometries espoused in Felix Klein's Erlangen programme (1872), not just geometry as defined by the transformations under which Euclidean figures in the plane remain unchanged. But there are also chapters that consider less geometrical topics. The construction of and the search for relationships among things may operate in space more abstracted from perception and conception than even geometry.

'Geometry' is a word of Greek origin meaning 'land measurement'[2] or 'earth measure'.[3] The Greek historian Herodotus (*c.* 485–425 BC) attributed the origin of geometry to the Egyptians' apportionment of land by equal measure and the relationship of the revenue claimed by the pharaoh according to the land area and any reduction in that area, duly measured by the pharaoh's overseers, during the annual flooding of the Nile. So geometry is considered to be descended from the concrete subdivision and organization of space. In this sense, architecture and geometry are mutually implicated in their conception and development. Both have the power to express and organize space using concepts outside the constraints of a direct mapping to a physical representation. The principal distinction lies in their levels of abstraction and generality. Geometry looks for generalities and, once established (demonstrated or proved), offers them up for use; architecture employs these general relationships constructively to underpin and create specific spatial relationships. It is assumed that practical geometry has been practiced since prehistoric times (in building, for example), but that the Greeks took it further, abstracting and systemizing it and leaving us with Euclid's axioms. Geometrical knowledge developed and crystallized.[4]

'Arithmetic' is also from the Greek, derived from the word αριθμός, meaning number. It is clear that the concept of number is deep in the human psyche; as architectural theorist Sanford Kwinter reminded us, Alfred North Whitehead observed that the day a connection was forged in the human mind between seven fish in a river and seven days, a landmark advance was achieved in the history of thought.[5] Near newborn infants are found to distinguish between one, two and three objects (a process known as subitizing), and we are still quite young when three or more is discretized into more precise denominations. Mathematics calls on conceptual metaphor to locate these numbers on a line, which gives us cognitive access to negative numbers, and numbers in between numbers – the real numbers that, as Kwinter points out, are anything but.[6] By the time we have engaged addition, subtraction, multiplication and division, the natural numbers that can be equated with fish and days are already found wanting. Zero, negative numbers, fractions, irrationals, and – more recently – real and imaginary numbers gradually, over millennia, find their way into the lexicon as part of the conceptual landscape and, in most (but not all) cases, onto the number line. This continuity of development is there to see in the imprint of the Babylonian sexagesimal system, evident in our measurement of time in groups of 60 seconds and 60 minutes, or angles of rotation with 360° to a full rotation. Notation has been a key innovation in the development of modern algorithms; the adoption of Arabic numerals with their use of decimal place ordering has reduced the weighty tomes needed to describe quite rudimentary mathematical relations, expressed in language in the manner of the Greeks.

What is the relationship of numbers to architecture? Numbers give us dimensions and proportions, fix the geometry as shape, and inscribe sacred meaning in significant buildings. And what of algebra, the third constituent of mathematics? Ingeborg M. Rocker, in his article 'When Code Matters', has this to say: 'Today, when architects calculate and exercise their thoughts, everything turns into algorithms! Computation, the writing and rewriting of code through simple rules, plays an ever-increasing role in architecture.'[7] Algebra is the primary meta-language of mathematics, in which both geometrical objects and numbers are further abstracted and generalized. The formal (logical) languages of computer code are not algebra, but algebra has provided the language in which to couch all the spatial, proximal and numerical relationships the algorithm-writing architect has in play. Its development marked the beginning of a progression to the ever-higher levels of abstraction and generalization that continue to empower mathematicians over new dominions of thought today.

What, therefore, is the philosophical impulse, the aspect of mathematical thought, that has excited the architectural activity of recent years, and to what is it reacting? The empirical experiments in perspective of the Renaissance artist and architect drove subsequent developments in projective geometry. By contrast, the archetypal modern-day architect has been seen as a mathematical reactionary, distant from the revolutions in geometry and newly possible understandings of space of the 19th and 20th centuries. In the late 1920s, architecture, in embracing one idea of industrialized production, sidelined the more expressionist, nationalistic and biologically inspired streams of Modernism to espouse the aesthetic dogma of the International Style, the pre-Renaissance interest in Vitruvius' proportions of the human body, Platonic interpretations of the Cosmos, geometrical ordering of the Euclidean plane, and the certainties of Cartesian space. The size, homogeneity and power of this idea, grounded in the ancient philosophical tradition of the search for absolute truth, has been so robust that the Postmodern vanguard movements in architecture that followed have tended to be just that: critiques of Modernism. Whether ironic, 'effetely Derridean or ponderously Tafurian', the critical practices of the vanguard prospered on the certainties of Modernism – on ideas, theories and concepts given in advance.[8] In relation to mathematics, architect and historian Robin Evans referred to the architectural use of 'dead' geometries.[9]

Outside this critical framework, the imagination of many leading architectural figures was caught by the chaotic systems of Edward Norton Lorenz and the fractal geometries of Benoît Mandelbrot, whose essay 'Fractals: Form, Chance and Dimension' (1975) provided the main conduit from complexity science to architecture. The three destabilizing ideas of discontinuity, recursivity and self-similarity were subsequently taken up by a string of architects as ideas for organizing principles.[10] But by the 1990s, there was already a trend to deny the inspiration of chaos, and to disown a fashion seen as having little tangible connection to the central concerns of architectural production. It was, in effect, an idea before its time. Given very few years and the computational means with which to explore the generative potential of recursive systems, these ideas re-entered architecture as if by stealth at a much more engaged level, and became part of the working design lexicon. Meanwhile, the distributed, networked and overlapping space of the post-20th-century human experience has brought connectivity into the foreground when considering spatial models, and given architects, as space-makers, an entrée into the conceptual spaces first defined by mathematicians and philosophers in the 19th century. Topology in architecture is no longer a critique of the power of the plane and gravitational vector in mainstream Modernism. Instead, it is the reality that we experience. Metrics and vectors have given place to distributed networks. This is a fundamentally different space in which to live.

What is the significance for architecture? Philosophically, Gilles Deleuze gave us the 'body', any corporeal arrangement composed of an infinite number of parts that are held together when they

move in unison at the same speed, more or less powerful, more or less able to effect change in their environment, depending on the degree to which they are capable of being affected themselves.[11] Michael Speaks argues that contemporary architectural practise as such a body becomes more powerful 'to the degree that it transforms the chatter of little truths into design intelligence'.[12] 'Intelligence' has become a word much overused in our time. It remains ill-defined, and in its current popular usage smacks of 'artificial intelligence', with the deliberate excision of the unpopular idea of artificial. The rise of the biological sciences has created a new atmosphere of respect for the living; now artificial systems are rated according to the extent to which they ape biological systems, a reversal of the situation that once pertained, whereby logic-based systems were looked to for insight into thought.

Dynamic, variable, spatial models are not new in spatial design, and are certainly not confined to electronic computation. But the power to integrate many variables, make links outside the confines of three geometrical dimensions, and simulate scenarios has given the concept of intelligent models new and more mainstream life. At the most general level, computing itself has had an undeniable influence on the mathematics and collective perception of human organization. At the same time, it promises to immerse architects into the very systems of complexity that had excited them metaphorically. It is bringing chaotic and unpredictable behaviour from the metaphorical to the operational sphere. Virtual models of emergent systems, parametric models that exhibit chaotic, even catastrophic, behaviour through their string of dependencies – in the last ten years there has been an architectural interest in formal systems that has been grounded theoretically only to the extent that modelling itself is grounded theoretically (that is, mathematically). This interest has offered up formal outcomes that were initially novel and are now recognizable, and a timely sense in which design, freed from Modernist doctrine, has been at liberty to explore newer, more diverse mathematical models. The stock-in-trade of mathematics is the useful generality, while design is concerned with the specific problem, whether related to site, programme, social context or technical issues. But currently there is a fierce contextual generality across design: the need for rapid and universal quantum reduction in consumption and environmental degradation. The mathematics–design nexus in its newly pluralist and agile manifestation is ubiquitous in this mission.

This book is about architecture, but not exclusively about architects. In many of the projects on the following pages, the design teamwork between different professions is explicit. There is a historical perspective here. Robin Evans, in his book *The Projective Cast*, gives a fine account of the geometrical baton passing from the architects to the engineers. The two building-design disciplines were hitherto divided along civil and military lines, but after the 18th century, and specifically the work of the French engineer Gaspard Monge, the knowledge of descriptive geometry that had been so vital but subsequently lost in the architecture profession was appropriated by engineers, reappearing in the great ship-building and railway projects of the 19th century.[13] Mathematical knowledge in construction design from this time onwards became more or less the exclusive domain of the engineer.

During the 20th century, built works outside the rectilinear dogma of Modernism are attributable in great number to notable engineering authorship, including Félix Candela, Pier Luigi Nervi, Heinz Isler and Frei Otto, to name a few. What is interesting about these engineers is their use of physical analogue models to find structurally efficient shapes that provide the general principle of the model as a responsive dynamic system. Moreover, in the last century there have been architect-led works that espoused the themes discussed in the chapters that follow, which were designed without the power of digital computation – seminal works that whisper of closer geometrical affinity to natural form. Why is Le Corbusier's chapel of Notre-Dame-du-Haut at Ronchamp absent from the discussion of surfaces? Even more pressingly, how can we discuss datascapes in the absence of the progenitor of all datascapes – the Philips Pavilion for Expo 58 – a parametrically conceived architecture driven by sound and manifest in an assembly of ruled surfaces? Undoubtedly, the partnership of Le Corbusier and Iannis Xenakis lead us by the soul to the essence of the poetics of mathematics in architecture. As with all books, a line has been drawn around the subject, and this one has corralled the widespread zeitgeist that access to electronic computational power, in tandem with accessible graphical interaction with virtual space, has brought to mathematical creativity in architecture.

Have architects regained mathematical literacy? Certainly there are more architects working in more abstract symbol-articulated spaces. Digital computation has been generous in its rewards, and the incentives for addressing the machine in its own powerful languages are significant. The profession is grappling with logic as never before. This has more significance than a mere change of protocol: it is a whole new spatial and temporal context for design that is free from the static Cartesian strictures of the two-dimensional drafting plane. It is a meta-zone of multiple simultaneous possibilities. The magnitude of the change is similar to the way the introduction of analysis and algebra has increased the curves or surfaces represented by a single, economical line of mathematical notation to a large and infinite number.

The subjects of each of the book's six chapters have been architecturally, rather than mathematically, selected, suggested by the works themselves. What all the projects have in common in the first chapter, 'Mathematical Surfaces and Seriality', is that the shape of their curved surfaces is a principal expressive element in the architecture. They are diverse in the nature of that surface and its creation. All use mathematical rules or technique. Here we see mathematics used to offer solutions to the 'problem' of defining and building free-form surfaces, a minimal surface used for its complex configuration and symbolic mathematical identity, hyperbolic surfaces substracted from the building mass, surfaces created through inversion in the sphere, toroid surfaces used for their rational tilable qualities, and surfaces shaped by gravity to solve the problem of structural minimalism.

The second chapter, 'Chaos, Complexity, Emergence', includes projects in which fractals and self-similarity are at the heart of their expression, and where the simple elements of a system result in intriguing emergent results at another scale. Work that is fundamentally digital in production is contrasted with that created using more traditional techniques. 'Packing and Tiling' has some overlap with the previous topic in its inclusion of fractal tilings, but also introduces the phenomenon of aperiodicity: tilings and three-dimensional space-filling packings that do not map to themselves when translated. This contrasts with traditional Islamic tiling, which, while it includes complex multiple symmetries, is generally periodic. The exploration of aperiodicity is a relatively recent area of mathematics that has been given architectural expression.

'Optimization' is perhaps the best example of the use of computation in architecture. It includes projects in which diverse optimization methods find structurally economical form, elegantly resolve the panellization of surfaces, minimize the size of structural members, and resolve the perfect acoustic space. In general, the computer is used in a variety of ways to search a defined solution space, and to sculpt material by responding to strain. 'Topology' is a collection of works that have been driven less by shape and more by relationships of proximity and connection. Some projects explicitly investigate the architectural possibilities of non-orientable surfaces, such as the Möbius band and the Klein bottle – one adopts knots and the other the multiple instances of a single, topological description of a space. The final chapter, 'Datascapes and Multi-dimensionality', is a rare collection of interactive spaces, ranging from installations to urban design, which respond to stimuli from approaching humans to atmospheric change. These are genuinely multi-dimensional spaces by any mathematical definition that test Theo van Doesburg's ninth proposition in his manifesto, 'Towards a Plastic Architecture', published in *De Stijl* in 1924: '. . . with the aid of calculation that is non-Euclidean and takes into account the four dimensions, everything will be very easy.'[14]

Reviewing the projects presented, there is a natural division between those in which the primary mathematical constituent is an idea, and those where mathematics is first and foremost positioned as a problem-solver. In some, the two roles are balanced or combined, and in all, the mathematical idea or problem-solver is also instrumental in the design process and to the form of the architectural outcome. In all this activity, there is no real evidence of convergence between architect and mathematician, but there is a sense in which mathematics and mathematical ideas have contributed to the formation and cohesion of diverse creative teams. In 2002, Lionel March, a pioneer in the study of mathematical applications in architecture using computation, published an essay entitled 'Architecture and Mathematics Since 1960', a review of work in which March himself had been involved and that encompassed a range of research from land use and built-form studies to more qualitative investigations of form based on group theory of symmetries.[15] He cited educationalist Friedrich Fröbel, who observed that relational studies of form were 'forms of life'. In recent years, we have seen a steep uptake curve of the application of mathematical thinking in architecture, as this theoretically grounded work is followed by a period in which the mathematics has indeed come to life in the practice of architecture at every creative level. We wait, breathless, to see how this will play out in the working relationships and built environment in the years to come.